

Formation of deuterons by coalescence: Consequences on the deuteron number fluctuations

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Two scenarios for cluster production have since long been discussed in the literature: i) direct emission of the clusters from a (grand canonical) thermal source or ii) subsequent formation of the clusters by coalescence of single nucleons. While both approaches have been successfully applied in the past it has not yet been clarified which of the two mechanisms dominates the cluster production. We propose to use recently developed event-by-event techniques to study particle multiplicity fluctuations on nuclear clusters and employ this analysis to the deuteron number fluctuations to disentangle the two production mechanisms. We argue that for a grand canonical cluster formation, the cluster fluctuations will follow Poisson distribution, while for the coalescence scenario, the fluctuations will strongly deviate from the Poisson expectation. We estimate the effect to be 10% for the variance and up to a factor of 5 for the kurtosis of the deuteron number multiplicity distribution. Our prediction can be tested in the beam energy scan program at RHIC as well as experiments at the FAIR and NICA facilities.

I. INTRODUCTION

The formation of bound states in nuclear collisions has been investigated experimentally and theoretically for many decades. On the experimental side the fragmentation of the source has been studied extensively at very low energies to extract information about the nuclear liquid-gas phase transition [1, 2], while on the theoretical side those observations were studied in detail by [3–6]. Already very early it was pointed out that deuteron production can be used to infer thermodynamic properties of the system, e.g. the entropy per baryon [7]. At higher energies around $\sqrt{s_{NN}} = 3 - 20$ GeV, the data become more scarce. Here, the AGS experiments have provided data on the formation of clusters up to Helium [8], while the SPS experiments have measured data on clusters up to mass three [9, 10] even for the anti-particle sector [11]. Currently the RHIC and LHC experiments have also shown data on light (anti-)nuclei production at the highest energies, see e.g. [12, 13]. Theoretically, the mechanism for cluster production at intermediate and high energies is not well understood. It is clear that clusters do not stem from a break-up of the initial target and projectile nuclei but have to be formed newly towards the end of the fireball evolution. However, it is not a priori clear, if the clusters are directly formed at the chemical freeze-out, e.g. from a grand canonical thermal ensemble [14], or if the formation of the clusters happens at the kinetic surface by coalescence of individual nucleons [15]. While one may argue that lightly bound clusters (e.g. deuteron has a binding energy of only a few MeV) may either not be formed at the chemical freeze-out due to the large temperature and may be easily destroyed (if formed earlier) in the kinetic stage, the predictions

of clusters multiplicities within the statistical approach provide a good description of the measured data. On the other hand coalescence of neutrons and protons to deuterons after the kinetic freeze-out is certainly another possible process and allows to describe the experimental data equally well [15].

In this paper we propose to use the fluctuations of the deuteron number to distinguish between the two production/formation mechanisms. These studies have become possible due to the increased experimental possibilities available at RHIC BES.

II. SET UP

To elucidate the idea we compare three scenarios: i) Direct deuteron production from a grand canonical thermal ensemble at the chemical freeze-out. Here, all fluctuations are Poissonian and the scaled moments of the deuteron distribution σ^2/λ , $S\sigma$, and $\kappa\sigma^2$ are all unity. In addition, there is no correlation between the proton and the deuteron number. ii) Only production of nucleons from the grand canonical thermal ensemble at the chemical freeze-out is assumed, while deuterons are formed by coalescence after the kinetic freeze-out. In this case we consider two variations of the coalescence prescription: ii.a) The initial number of protons fluctuates according to a Poisson distribution, while the number of deuterons depends on the squared proton density. ii.b) Both proton and neutron fluctuations are Poissonian and the number of deuterons is proportional to their product. As a result in both coalescence scenarios the deuteron number will not fluctuate according to a Poisson distribution and all higher moments will show strong deviations from their

Poisson values.

Next we closer explain both used coalescence models.

A. Model A: Correlated proton and neutron number

To quantify the fluctuations in scenario ii.a) we follow the standard coalescence approach, i.e. deuterons are formed after the kinetic freeze-out in each event with a probability proportional to the squared number of all initially produced protons [15, 16], i.e.

$$\lambda_d = B n_i^2, \quad (1)$$

where B denotes the coalescence parameter, which may depend on the center-of-mass energy. In this model we actually make the (standard) assumption that neutron yield is directly correlated with the proton yield in that event. (This assumption will be relaxed in the other model.) The strength of the event-by-event proton-neutron correlation can be extracted from experimental measurements, see also Appendix. We emphasize that the predicted fluctuation signal should be studied in a fixed volume, e.g. using tight centrality cuts to avoid volume fluctuations.

The number of deuterons n_d in a given reaction with fixed initial proton number n_i is then given by a Poisson distribution

$$P_d(n_d|n_i) = \lambda_d^{n_d} \frac{e^{-\lambda_d}}{n_d!} = (B n_i^2)^{n_d} \frac{e^{-B n_i^2}}{n_d!}. \quad (2)$$

Summing over the initial proton numbers distributed according to $P_i(n_i)$ then leads to the final deuteron number fluctuations based on the initial proton number fluctuations as

$$P_d(n_d) = \sum_{n_i \geq n_d} P_d(n_d|n_i) P_i(n_i). \quad (3)$$

Recall that $P_i(n_i)$ is Poissonian and the initial proton number can be obtained as

$$n_i = n_p + n_d \quad (4)$$

where n_p is the mean *observed* proton number.

B. Model B: Independent proton and neutron fluctuations

The coalescence model presented in previous subsection is extreme in its assumption that neutron and proton fluctuations are strongly correlated. In order to make robust predictions for the moments of the deuteron number distribution we will relax the assumption completely and consider the model with independent proton and neutron fluctuations. The initial proton (neutron) number n_i (n_j) fluctuates again according to Poisson distribution

and the deuteron formation probability is proportional to the product of nucleon multiplicities, i.e.

$$\lambda_d = B n_i n_j. \quad (5)$$

The coalescence parameter B depends again only on the collision energy. The initial neutron number n_j fluctuates according to a Poisson distribution with the same mean number as the initial proton number.

The number of deuterons in events with given number of nucleons is given by the Poisson distribution

$$P_d(n_d|n_i, n_j) = \lambda_d^{n_d} \frac{e^{-\lambda_d}}{n_d!} = (B n_i n_j)^{n_d} \frac{e^{-B n_i n_j}}{n_d!}. \quad (6)$$

The deuteron number distribution is then obtained by summing up over the initial proton and neutron number distributions

$$P_d(n_d) = \sum_{n_i, n_j \geq n_d} P_d(n_d|n_i, n_j) P_i(n_i) P_j(n_j). \quad (7)$$

It has been shown recently that if deuteron production scales with the squared proton number (Model A) then higher scaled moments of the observed proton number fluctuations agree qualitatively with the data from RHIC beam energy scan program [17, 18]. Unfortunately, this feature does not survive in Model B. Nevertheless, we also use it in our calculation. The real situation is perhaps somewhere in between the two used models and by showing that our signal is clearly visible in both cases we will prove its robustness.

III. MOMENTS OF THE DEUTERON DISTRIBUTION

The essential model parameter is the coalescence parameter B which we fix to obtain the correct mean deuteron multiplicity at midrapidity for each energy. While the data on proton multiplicities at midrapidity are abundant, the data on deuterons are not available for all the examined energies. The available data are however well reproduced by the thermal model. Deuteron to proton ratio d/p can be thus parametrized as:

$$\frac{d}{p} = 0.8 \left[\frac{\sqrt{s_{NN}}}{1 \text{ GeV}} \right]^{-1.55} + 0.0036. \quad (8)$$

The observed proton and parametrized deuteron multiplicities are well reproduced in our model. We summarize the model parameter B and results for the mean values of proton and deuteron multiplicities for various collision energies in Fig. 1. Plotted are values for Model A. The difference to Model B is so small that if it was also plotted the data points would practically overlap.

For illustration, Fig. 2 shows the distributions of deuteron number for Au+Au collisions at 2.6 GeV beam energy in comparison to the Poisson distribution. Here

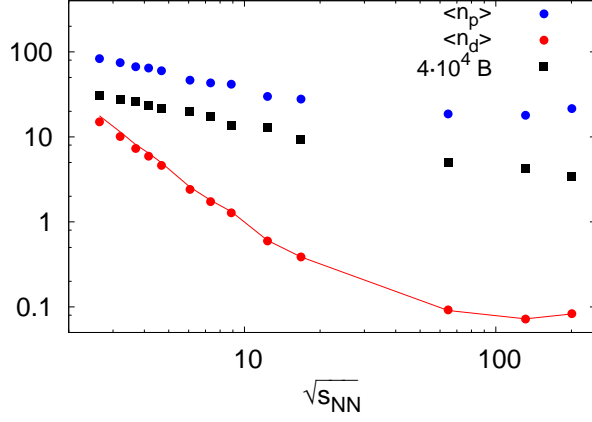


FIG. 1. Model parameter B for Model A (black squares) and resulting proton (in blue) and deuteron (in red) multiplicities as function of energy. The resulting deuteron multiplicity is compared to the thermal fit (red line) input to our model.

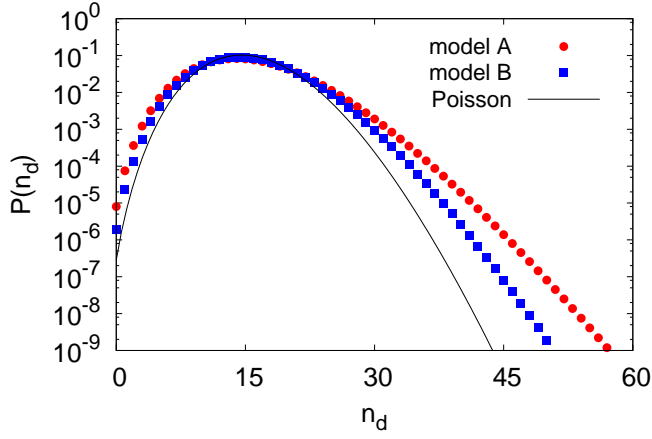


FIG. 2. Fluctuation of the deuteron number for Au+Au collisions at 2.6 GeV beam energy in comparison to the Poisson distribution. The parameters of the distributions are for Model A: $\sigma^2/\langle n_d \rangle = 1.609$, $S\sigma = 2.218$, $\kappa\sigma^2 = 6.915$; Model B: $\sigma^2/\langle n_d \rangle = 1.308$, $S\sigma = 1.616$, $\kappa\sigma^2 = 3.422$.

one clearly observes that coalescence leads to skewed distributions with a shift to higher values, as expected from the non-linear formation probability. The scaled higher moments: the variance $\sigma^2/\langle n_d \rangle$, the skewness $S\sigma$ and the kurtosis $\kappa\sigma^2$ all differ significantly from the Poisson expectation of unity. The departure from Poissonian distribution is larger if proton and neutron number fluctuate together (Model A), but also independent proton and neutron fluctuations (Model B) lead to clearly non-Poissonian shape.

Next we explore the energy dependence of the moments of the deuteron distribution and compare to the Poisson expectations. Figures 3 and 4 show the scaled moments $\sigma^2/\langle n_d \rangle$, $S\sigma$, and $\kappa\sigma^2$ as functions of collision en-

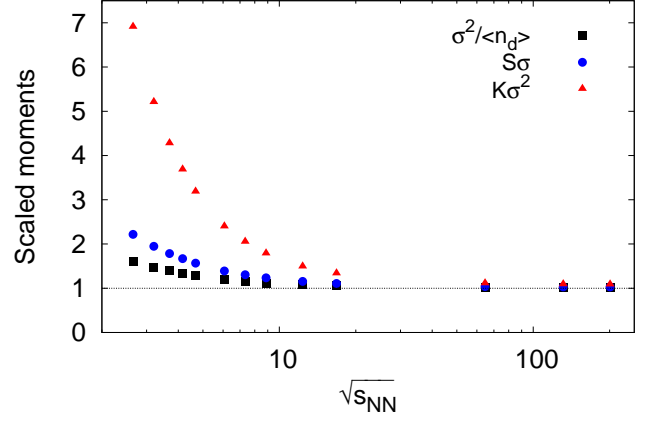


FIG. 3. The energy dependence of the moments $\sigma^2/\langle n_d \rangle$, $S\sigma$, and $\kappa\sigma^2$ of the deuteron distribution obtained from Model A compared to the Poisson expectation

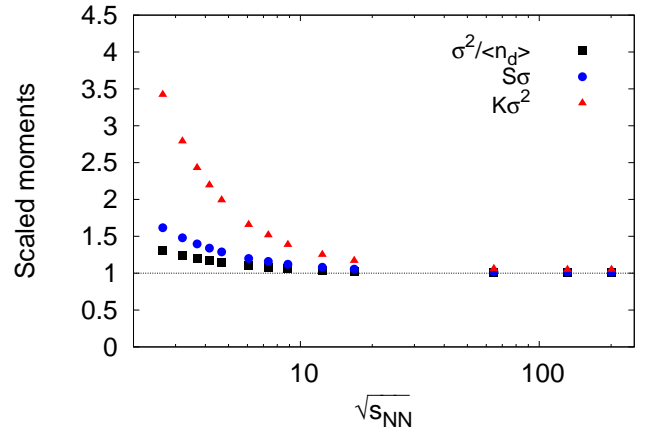


FIG. 4. The energy dependence of the moments $\sigma^2/\langle n_d \rangle$, $S\sigma$, and $\kappa\sigma^2$ of the deuteron distribution in the coalescence model assuming independent proton and neutron fluctuations (Model B) compared to the Poisson expectation.

ergy for Models A and B, respectively. We observe a clear deviation from the Poisson expectation for all the higher moments. The deviation is very strong at low energies, where both coalescence parameter B and the mean proton and neutron numbers are large, which results in sizeable fluctuations of the mean of Poissonian deuteron number distribution given by eq. (1) or eq. (5). This leads to even larger fluctuations of the deuteron number. The effect could be possibly observed for energies up to about 5 GeV in all the moments and even up to higher energies in kurtosis only.

Removing the correlation between initial proton and neutron fluctuations clearly weakens the effect, as can be seen in Fig. 4. The scaled moments attain approximately one half of the values obtained for neutron num-

ber coupled to the proton number. We can still conclude, however, that we should be able to observe the deviation of the moments of the deuteron multiplicity distribution in case of coalescence regardless of the strength of the correlation of nucleon fluctuations. Additional information about the degree of this correlation can be provided by proton-deuteron multiplicity correlations, as shown in the Appendix.

IV. SUMMARY

In the light of the ongoing debate about the origin of the deuterons we propose to measure data on fluctuations of deuterons. Employing standard approach and using the deuteron formation probability as proportional to the square of the nucleon yield we obtain strongly non-Poissonian distribution of the deuteron yield. Exact shape of deuteron distribution depends on whether proton and neutron yield are correlated or not. We tested both extremes—i.e., strongly correlated and completely independent—and the departure from Poissonian is always large. This allows to disentangle the direct grand canonical production of deuterons (and other clusters) from the formation of deuterons by coalescence.

For simplicity, we have assumed that the initial proton number follows a Poisson distribution. This is fine for measurements with small acceptance. If baryon number is exactly conserved within the acceptance window, then the initial proton number should follow binomial distribution. Nevertheless, all our results should not change qualitatively. Note also, that direct comparison with experimental data will also have to include the fluctuations of volume.

Our predictions are testable by the current experiments at the RHIC-BES and later by FAIR.

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Appendix: Correlations

Another observable which distinguishes thermal grand-canonical production and cluster production via coales-

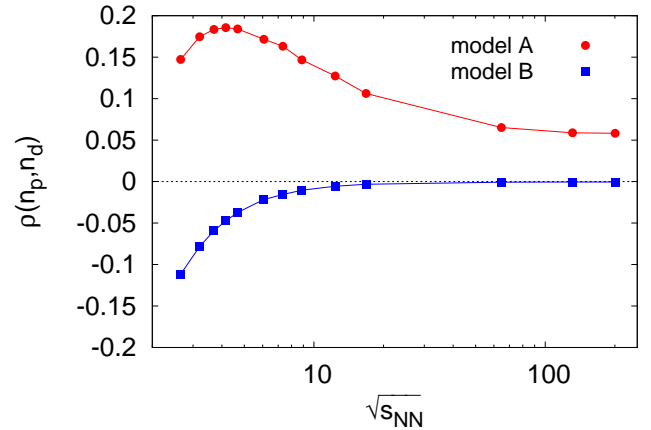


FIG. 5. Energy dependence of the correlation coefficients between proton number and deuteron number in the case of coalescence with strongly correlated proton and neutron fluctuations (Model A) and in the case of coalescence with independent proton and neutron fluctuations (Model B).

cence is the correlation of proton and deuteron multiplicities. In the grand-canonical statistical approach the proton and deuteron multiplicities both fluctuate independently according to the Poisson distribution. However, in the coalescence scenario the deuteron fluctuations are connected to the initial proton and/or neutron number and thus to the observed nucleon number fluctuations, as well. This leads to positive correlation between proton and deuteron multiplicities. On the other hand at a fixed initial proton number n_i a larger deuteron multiplicity n_d results in a smaller final proton number $n_p = n_i - n_d$, which introduces anticorrelation between n_p and n_d .

To explore to what degree the proton and deuteron multiplicities are correlated we evaluate the correlation coefficient ρ defined as

$$\rho(n_p, n_d) = \frac{\sum_k (n_{pk} - \lambda_p)(n_{dk} - \lambda_d)}{\sigma_p \sigma_d}, \quad (\text{A.1})$$

where $\lambda_p(\lambda_d)$ and $\sigma_p(\sigma_d)$ are the mean value and width of proton (deuteron) multiplicity distribution. The mean value and the width for both protons and deuterons are calculated using the distributions we derived earlier.

Now we have to distinguish between the two coalescence models we introduced. First, we explore Model A with strongly correlated initial proton and neutron fluctuations. To calculate the correlation coefficient consistently within this approach we sum over all possible (n_i, n_d) states:

$$\rho(n_p, n_d) = \frac{\sum_{n_i, n_d} P(n_i) P(n_d | n_i) (n_i - n_d - \lambda_p)(n_d - \lambda_d)}{\sigma_p \sigma_d}. \quad (\text{A.2})$$

In Fig. 5 the correlation parameter $\rho(n_p, n_d)$ is shown as function of energy. The proton and deuteron multiplicity

are positively correlated. The correlation is stronger for lower energies, where the coalescence parameter is larger. An interesting feature of the results is the decrease of the correlation for the lowest energies. This effect is caused by the wide deuteron distribution which emphasizes the extreme low and high values of deuteron number leading to stronger anticorrelation. Overall correlation is thus reduced.

Next we will investigate the correlations in the coalescence model with independent initial nucleon fluctuations (Model B). In this case, we calculate the correlation coefficient by summing over all possible (n_i, n_d, n_j) states:

$$\rho(n_p, n_d) = \frac{1}{\sigma_p \sigma_d} \times \sum_{n_i, n_d, n_j} P(n_i) P(n_j) P(n_d | n_i, n_j) \times (n_i - n_d - \lambda_p)(n_d - \lambda_d). \quad (\text{A.3})$$

The correlation parameter $\rho(n_p, n_d)$ calculated assuming independent proton and neutron fluctuations is shown in

Fig. 5 as function of energy and compared with the result of Model A. There is a marked difference between the two models. The proton and deuteron multiplicities are now anticorrelated. The anticorrelation is caused by an interplay of two effects. Firstly, the number of deuterons now depends only linearly on the number of protons, as compared to the squared proton yield in Model A. Thus, the correlation is weaker. On the other hand, the anticorrelation is reinforced by the neutrons fluctuating independently, that cause an increase of higher deuteron production when the proton yield is lower, but neutron yield is high and vice-versa.

The proton-deuteron multiplicity correlation measurement yields very different results for the two coalescence scenarios. However, in both scenarios we see a clear deviation from a Poisson-like uncorrelated thermal proton and deuteron production. We propose this measurement as complementary to the moments of the deuteron distribution that could possibly help to shed some light on the degree of correlation in the nucleon production.

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- [1] A. D. Panagiotou, M. W. Curtin, H. Toki, D. K. Scott and P. J. Siemens, Phys. Rev. Lett. **52**, 496 (1984).
 - [2] J. Pochodzalla [ALADIN Collaboration], in *Proceedings of Hot and dense nuclear matter, Bodrum, 1993*, pp. 39-51.
 - [3] L. P. Csernai, in *Proceedings of High Energy Heavy Ion Collisions and Quark Degrees of Freedom in Nuclei, Puri, 1987*, pp. 69-131.
 - [4] S. Pratt and M. B. Tsang, Phys. Rev. C **36**, 2390 (1987).
 - [5] J. Aichelin, A. Bohnet, G. Peilert, H. Stoecker, W. Greiner and A. Rosenhauer, Phys. Rev. C **37**, 2451 (1988).
 - [6] J. Aichelin, Phys. Rept. **202**, 233 (1991).
 - [7] J. I. Kapusta, Phys. Rev. C **29**, 1735 (1984).
 - [8] S. Wang *et al.* [EOS Collaboration], Phys. Rev. Lett. **74**, 2646 (1995).
 - [9] I. G. Bearden *et al.* [NA44 Collaboration], Nucl. Phys. A **661**, 387 (1999).
 - [10] B. Baatar *et al.* [NA49 Collaboration], Eur. Phys. J. C **73**, no. 4, 2364 (2013) doi:10.1140/epjc/s10052-013-2364-3 [arXiv:1207.6520 [hep-ex]].
 - [11] M. Weber *et al.* [NA52 Collaboration], J. Phys. **G28**, 1921 (2002).
 - [12] C. Adler *et al.* [STAR Collaboration], Phys. Rev. Lett. **87**, 262301 (2001) [Erratum-ibid. **87**, 279902 (2001)] [arXiv:nucl-ex/0108022].
 - [13] J. Adam *et al.* [ALICE Collaboration], arXiv:1506.08951 [nucl-ex].
 - [14] A. Andronic, P. Braun-Munzinger, J. Stachel and H. Stoecker, Phys. Lett. B **697**, 203 (2011) [arXiv:1010.2995 [nucl-th]].
 - [15] J. L. Nagle, B. S. Kumar, D. Kusnezov, H. Sorge and R. Mattiello, Phys. Rev. C **53**, 367 (1996).
 - [16] S. T. Butler and C. A. Pearson, Phys. Rev. **129**, 836 (1963).
 - [17] M. M. Aggarwal *et al.* [STAR Collaboration], Phys. Rev. Lett. **105**, 022302 (2010).
 - [18] Z. Fecková, J. Steinheimer, B. Tomášik and M. Bleicher, Phys. Rev. C **92**, no. 6, 064908 (2015) doi:10.1103/PhysRevC.92.064908 [arXiv:1510.05519 [nucl-th]].